

BEE - UNIT-01 and UNIT-02

Derivations & Theorem

Lecture-08

classmate

Date

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SRIJEEV - 1008

FEERRCDM - Mag

* KVL

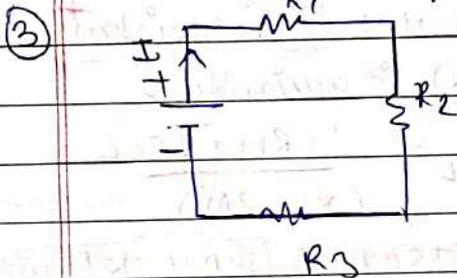
① In any closed loop, the algebraic sum of all branched voltages is zero.

$$\sum V + \sum IR = 0$$

$$V - IR_1 - IR_2 - IR_3 = 0$$

② Sign convention

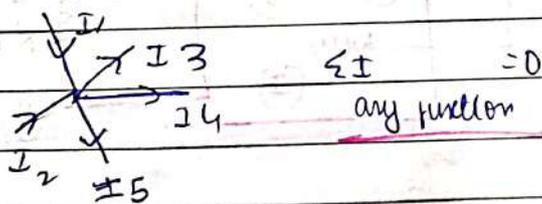
while tracing any path, the rise in potential is taken +ve and if there is a drop in potential then it is taken -ve.



$$V - IR_1 - IR_2 - IR_3 = 0$$

* KCL

① The algebraic sum of all currents meeting at any junction is always zero.



$$I_1 - I_2 + I_3 - I_4 - I_5 = 0$$

② Sign convention -

incoming current flowing towards +ve, outgoing -ve.

* SUPERPOSITION THEOREM

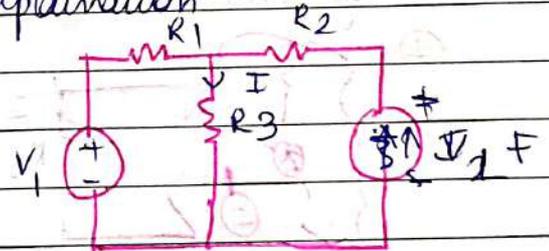
① In any linear bilateral active network containing,

→ two or more sources and many resistances, the current through any branch is the algebraic sum of all such currents which would be given by algebraic sum of individual currents flowing through that branch, when only one source is active at a time, and other sources are replaced by their internal resistance.

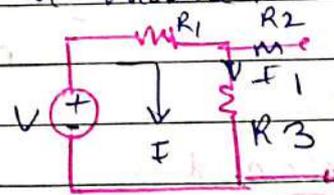
$V \rightarrow R_{int} = 0$ short

$I \rightarrow R_{int} = \infty$ open ckt

② explanation

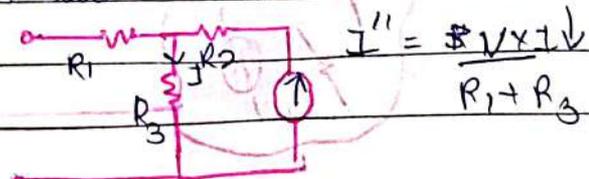


∴ a.o.v active



$$I' = \frac{V \times I}{R_1 + R_3}$$

b. I active



$$I'' = \frac{I \times I}{R_1 + R_3}$$

* THEVENIN'S THEOREM

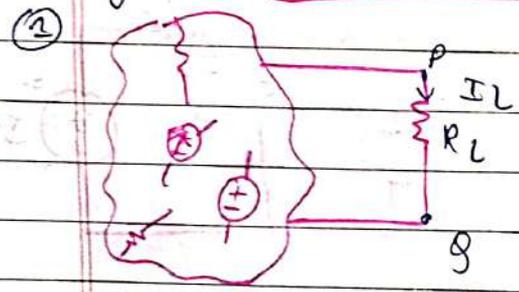
In any linear, bilateral active network, the current flowing through the load resistance is given by

$$I_L = \frac{V_{TH}}{R_L + R_{TH}}$$

where V_{TH} is the open circuit voltage across the terminals PQ with R_L removed.

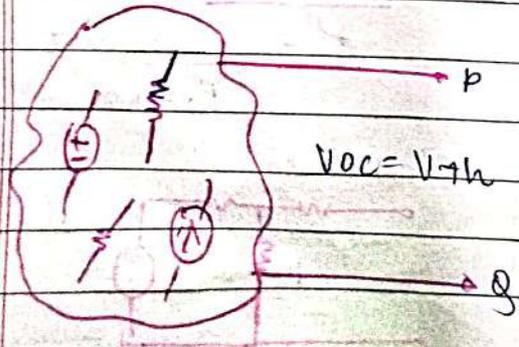
And R_{TH} is the equivalent resistance of the network across load terminals PQ with R_L removed.

And all sources are replaced by their internal resistance.

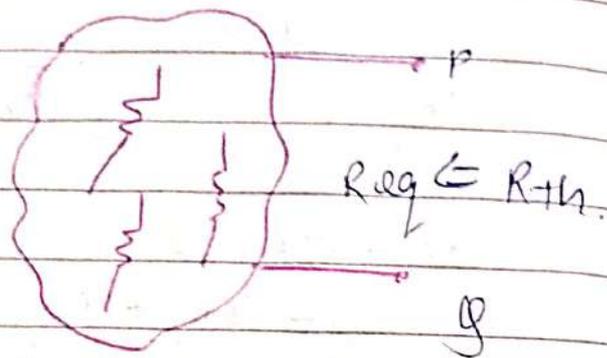


Network $I_L = \frac{V_{TH}}{R_L + R_{TH}}$

(2) R_L Removed



(3) Thevenin's Resistance



sources are replaced by internal resistance.

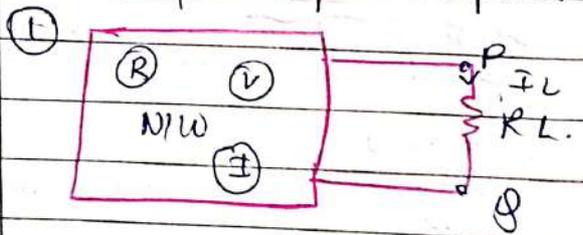
* NORTON'S THEOREM

In any linear, bilateral active network, the current flowing through the load resistance R_L (PQ) is given by:

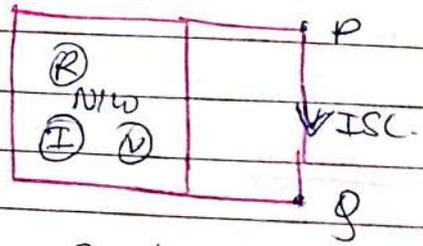
$$I_L = \frac{R_N \cdot I_{SC}}{R_L + R_N}$$

where I_{SC} is short circuit current when R_L is removed.

And R_N is the equivalent resistance across PQ with R_L removed and all the sources are replaced by their internal resistance.

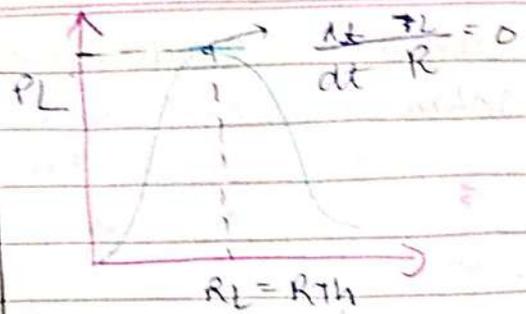
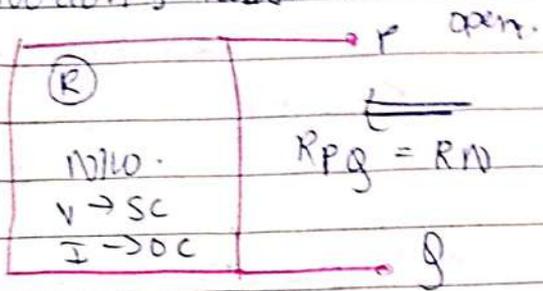


(2) R_L removed, short PQ terminal



short-open.

③ Norton's Resistance.



* CONDITION FOR (P_L) max.

$I = \frac{V_{TH}}{R_L + R_{TH}}$

$R_L = R_{TH}$

$P_L = P_{Lmax}$

$\frac{dP_L}{dR_L} = 0$

$\frac{d}{dR_L} \left[\left(\frac{V_{TH}}{R_L + R_{TH}} \right)^2 \times R_L \right]$

$(V_{TH})^2 \frac{d}{dR_L} \left[\frac{R_L}{(R_{TH} + R_L)^2} \right]$

$R_L^2 (R_{TH} + R_L) - (R_{TH} + R_L)^2 \times 1$
 $(R_{TH} + R_L)^2$

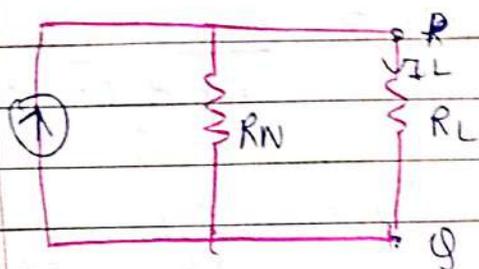
$2R_L (R_{TH} + R_L) - (R_{TH} + R_L)^2 = 0$

$2R_L (R_{TH} + R_L) = (R_{TH} + R_L)^2$

$2R_L = R_{TH} + R_L$

$R_L = R_{TH}$

④ Norton's Network



$I_L = \frac{I_{SC} \times R_N}{R_L + R_N}$

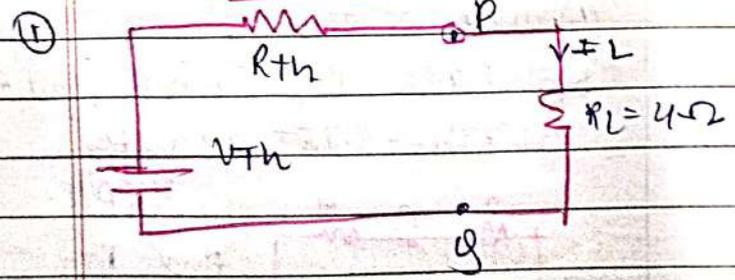
* MAX POWER TRANSFER THEOREM

In any linear, bilateral active network, the POWER transferred to the load resistance R_L is maximum when load

resistance is equal to Thevenin's resistance.

when $R_L = R_{TH}$

$P_L = P_L(max)$



* DEFINITION - RTC

• Is a measure of how much a material's resistance changes with temperature.

• Change is R per degree of Temp expressed in ohm / per degree C.

• $+TCR \rightarrow R \propto T$

• $-TCR \rightarrow R \propto \frac{1}{T}$

subtract

$$\frac{\alpha_2}{\alpha_1} \Delta t$$

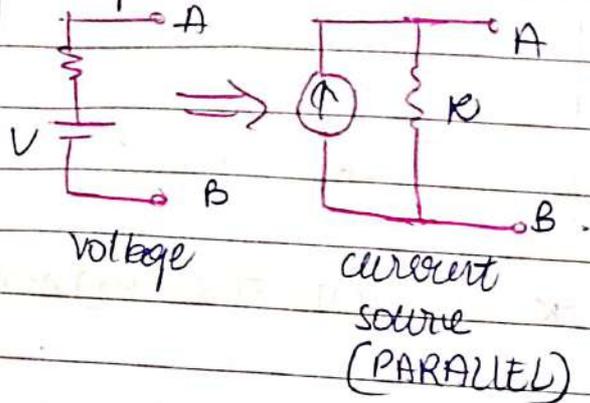
$$\Delta t = t_2 - t_1$$

$$-\Delta t = \frac{1}{t_1 - t_2}$$

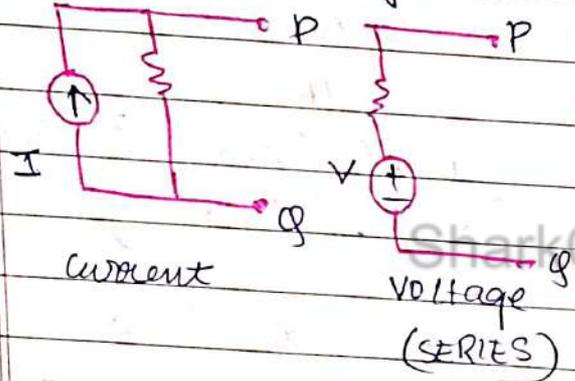
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SOURCE TRANSFORMATION

① Voltage to current source



② Current to voltage source



$$\frac{1}{1 + \alpha_1 \Delta t} = \frac{1}{1 + \alpha_2 \Delta t}$$

$$\alpha_1 \Delta t = \frac{1}{1 + \alpha_2 \Delta t}$$

$$\alpha_1 \Delta t = \frac{1 - \alpha_2 \Delta t}{1 + \alpha_2 \Delta t}$$

$$\alpha_1 = \frac{\alpha_2 \Delta t}{(1 + \alpha_2 \Delta t) \Delta t}$$

$$\alpha_{12} = \frac{\alpha_2}{1 + \alpha_2 \Delta t}$$

$$\alpha_1 = \frac{\alpha_2}{1 + \alpha_2 (t_1 - t_2)}$$

RTC DERIVATION

$$\Delta R = (R_2 - R_1) \Omega$$

$$(R_2 - R_1) \propto (t_2 - t_1)$$

$$(R_2 - R_1) \propto (R_1 (t_2 - t_1))$$

$$R_2 - R_1 = \alpha R_1 (t_2 - t_1)$$

$$\alpha_1 = \frac{R_2 - R_1}{R_1 (t_2 - t_1)}$$

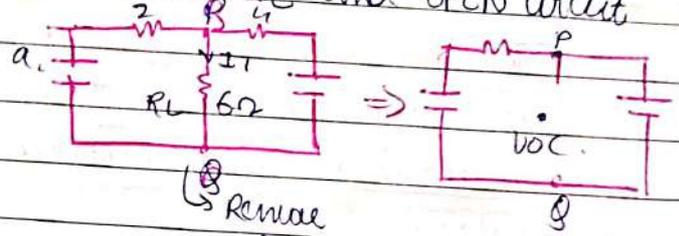
$$R_2 = R_1 [\alpha_1 \Delta t + 1]$$

$$R_1 = \frac{R_2}{\alpha_1 \Delta t + 1}$$

$$\frac{R_2}{R_1} = 1 + \alpha_1 \Delta t$$

SOLVING THEVENIN Questions

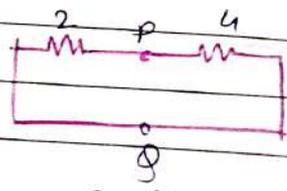
- ① Identify P and Q in RL
- ② Remove RL and OPEN circuit



③ Apply KVL in whole loop.

④ Find I and Voc.

⑤ Make Rth Thevenin, find Rth
b. Rth - SHORT all voltage supply.



Find Rth

By P/S of Resistance

Delta equations

- $R_{12} = (R_{12}) \times (R_{23} + R_{31})$
- $R_{12} = \frac{R_{12} \times (R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}}$

Star equations

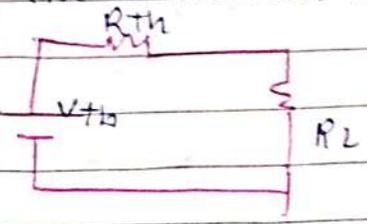
- $R_{12} = R_1 + R_2$
- $R_{23} = R_2 + R_3$
- $R_{31} = R_3 + R_1$

Final star equations

- $R_1 = \frac{R_{12} \times R_{31}}{R_{12} + R_{23} + R_{31}}$
- $R_2 = \frac{R_{23} \times R_{12}}{R_{12} + R_{23} + R_{31}}$

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c. Rth + Thevenin circuit



Make a thevenin circuit with R_{th} , R_L and V_{th} .

d. $I_L = \frac{V_{th}}{R_L + R_{th}}$

• $R_1 + R_2 = \frac{(R_{12}) \times (R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} \rightarrow (7)$

• $R_2 + R_3 = \frac{(R_{23}) \times (R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}} \rightarrow (8)$

• $R_3 + R_1 = \frac{(R_{31}) \times (R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} \rightarrow (9)$

Add (7) + (8)

• $R_1 + R_2 + R_2 + R_3 = \frac{(R_{12}) \times (R_{23} + R_{31}) + (R_{23}) \times (R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}}$

• $R_1 + 2R_2 + R_3 = \frac{R_{12}R_{23} + R_{12}R_{31} + R_{23}R_{31} + R_{12}R_{23}}{R_{12} + R_{23} + R_{31}}$

• $R_1 + 2R_2 + R_3 = \frac{2R_{12}R_{23} + R_{12}R_{31} + R_{23}R_{31}}{R_{12} + R_{23} + R_{31}} \rightarrow (10)$

• (10) - (9)
 $\Rightarrow R_1 + 2R_2 + R_3 - R_3 = R_1$
 $= \frac{2R_{12}R_{23} + R_{12}R_{31} + R_{23}R_{31} - R_{12}R_{23}}{R_{12} + R_{23} + R_{31}}$

$= \frac{R_{12}R_{31} + R_{23}R_{31}}{R_{12} + R_{23} + R_{31}}$
 $= 2R_2 = \frac{2R_{12}R_{23}}{R_{12} + R_{23} + R_{31}}$

- $R_{12} = R_1 + R_2 \rightarrow (1)$
- $R_{23} = R_2 + R_3 \rightarrow (2)$
- $R_{31} = R_3 + R_1 \rightarrow (3)$
- For Delta
- $R_{12} = \frac{(R_{12})(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}}$
- $R_{12} = \frac{(R_{12}) \times (R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} \rightarrow (4)$
- $R_{23} = \frac{(R_{23}) \times (R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}}$
- $R_{23} = \frac{(R_{23}) \times (R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}} \rightarrow (5)$
- $R_{31} = \frac{(R_{31}) \times (R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} \rightarrow (6)$

- $R_2 = \frac{R_{12} R_{23}}{R_{12} + R_{23} + R_{31}}$
- $R_3 = \frac{R_{31} \times R_{23}}{R_{12} + R_{23} + R_{31}}$
- $R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}$

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$$R_1 = \frac{R_{12} R_{31} + R_{31} R_{23}}{R_{12} + R_{23} + R_{31}}$$

STAR TO DELTA

- Add (1) + (5) + (6)
- Separate R_{12}
- Take $R_3 \rightarrow R_{12} \times R_3$

★ STAR TO DELTA

- write star equations
- $R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}} \rightarrow (1)$
- $\frac{R}{2} = \frac{R_{23} \cdot R_{12}}{R_{12} + R_{23} + R_{31}} \rightarrow (2)$
- $\frac{R}{3} = \frac{R_{31} \cdot R_{23}}{R_{12} + R_{23} + R_{31}} \rightarrow (3)$
- multiply $R_1 R_2, R_2 R_3, R_3 R_1$
- $R_1 R_2 = \frac{(R_{12} R_{31}) (R_{23}) (R_{12})}{(R_{12} + R_{23} + R_{31})^2}$
- $R_1 R_2 = \frac{(R_{12})^2 (R_{31}) (R_{23})}{(R_{12} + R_{23} + R_{31})^2} \rightarrow (4)$
- $\frac{R}{2} \frac{R}{3} = \frac{(R_{23})^2 \cdot (R_{31}) (R_{12})}{(R_{12} + R_{23} + R_{31})^2} \rightarrow (5)$
- $R_3 R_1 = \frac{(R_{31})^2 (R_{12}) (R_{23})}{(R_{12} + R_{23} + R_{31})^2} \rightarrow (6)$

• Add (4) + (5) + (6)

$$R_1 R_2 + R_2 R_3 + R_3 R_1$$

$$\Rightarrow \frac{R(R_{12})^2 (R_{31}) (R_{23}) + (R_{23})^2 \cdot (R_{31}) (R_{12}) + (R_{31})^2 (R_{12}) (R_{23})}{(R_{12} + R_{23} + R_{31})^2}$$

Separate multiplied & added one

$$\frac{(R_{12} R_{23} R_{31}) (R_{12} R_{23} R_{31})}{(R_{12} + R_{23} + R_{31})^2}$$

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{(R_{12})^2 R_{23} R_{31}}{R_{12} + R_{23} + R_{31}}$$

• Separate R_{12}

$$R_{12} \times \left[\frac{R_{23} \cdot R_{31}}{R_{12} + R_{23} + R_{31}} \right] = R_1 R_2 + R_2 R_3 + R_3 R_1$$

$$R_{12} \times R_3 = R_1 R_2 + R_2 R_3 + R_3 R_1$$

$$R_{12} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

$$R_{12} = \frac{R_1 R_2}{R_3} + \frac{R_2 R_3}{R_3} + \frac{R_3 R_1}{R_3}$$

$$R_{12} = R_1 + R_2 \cdot \frac{R_1 R_2}{R_3}$$

$$R_{23} = R_2 + R_3 \cdot \frac{R_2 R_3}{R_3}$$

$$R_{31} = R_3 + R_1 \cdot \frac{R_1 R_3}{R_2}$$

★ SOLVING Superposition Questions

- ① make 1st source active
 - write 2 KVL eqn's
 - Find I_1 and $I_2 = I'$
- ② make 2nd source active
 - write 2 KVL eqn.
 - Find I_1 and $I_2 = I''$
- ③ $I = I' + I''$

UNIT-03-MAGNETIC CIRCUITS.

* SOLVING NORTON'S QUESTIONS

- $I = \frac{ISC \times RN}{RL + RN}$
- ① Remove R_L and short PG to find ISC - short current
ISC = $I_1 - I_2$
KVE and find ISC.
- ② Norton resistance.
short all voltage source and find R_N .
- ③ Norton's circuit.
 R_N, I_L, R_L
find $I_L = \frac{ISC \times R_N}{R_L + R_N}$

- * ④ Delta To star
 - a. Star R_{12}, R_{23}, R_{31}
 - b. Delta - $R_{12} = \frac{R_{12}(R_{23} + R_{31})}{\text{Sum}}$
 - c. write same as addition $R_1 + R_2$
 - d. add ⑦ + ⑧
 - e. star equations \rightarrow
 $R_2 = \frac{R_{12} R_{23}}{\text{Sum}}$ common.

- ② Star to Delta
 - a. write ③ final star equations.
 - b. multiply $R_1 R_2 = \frac{(\text{square})^2 \times (x)}{R_2 R_3 \cdot (\text{Sum})^2}$

- ① add ④ + ⑤ + ⑥
- ② separate and let square in deno.

$$\frac{\text{Sum} \cdot (P_{\text{load}})}{(S)^2}$$

$$\hookrightarrow \frac{P_{\text{load}}}{\text{sum}}$$

- ③ separate $R_{12} \times \frac{R_{23} \cdot R_{31}}{\text{Sum}}$
 $\hookrightarrow R_3$

$$R_{12} \cdot R_3 = \text{LHS}$$

$$R_{12} = \frac{\text{LHS}}{R_3}$$

$$R_{12} = \frac{''}{R_3} + \frac{''}{R_3} + \frac{''}{R_3}$$

$$R_{12} = \frac{\text{load}}{R_3} \times \text{sum of } R_1 + R_2$$

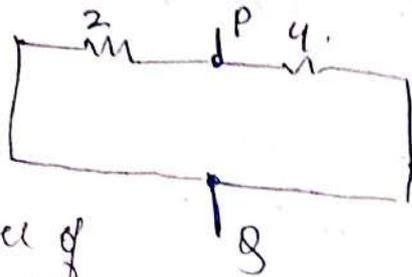
$$R_{11} = \frac{R_1 R_2}{R_3} \times R_1 + R_2$$

- * ① Thevenins - open PG
 - a. send current I in whole net, make one full loop
 - b. ignore Voc and make equation of current, get I ✓
 - c. Make Voc equation
 - d. Find Voc by putting value of I in equation.

Teacher's Signature

f. highlight the terminals P & Q and put only R in circuit

• Do S or P and find

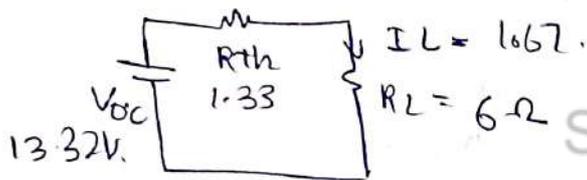


Parallel of 2 and 4

$$R_{th} = \frac{2 \times 4}{2+4} = 1.33 \Omega$$

g. Thevenin's Network

• Add R_{th} and R_L and I_L show in ckt.



$$I_L = \frac{V_{th}}{R_L + R_{th}}$$

get I_L ✓

② Norton's Problems

a. short P & Q and find ISC in ckt terminals

$$I_{SC} = I_1 - I_2$$

b. Make two separate equations for loop 1 and loop 2

c. ignore ISC and write KVL eqn

$$4I_2 + 2 = 0$$

$$2I_1 - 10 = 0$$

d. get I_1 and I_2 ✓

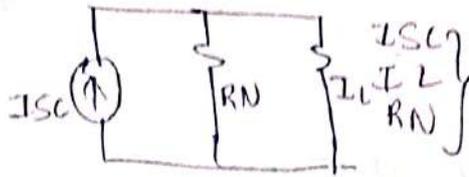
e. Find $I_{SC} = I_1 - I_2$ ✓

f. find R_N

make ckt with resistors

R_N ✓

g. Norton's Network



$$I_{SC} = I_1 - I_2 = \checkmark$$

h. Find I_L in terminals

$$I_L = I_{SC} \times \frac{R_N}{R_L + R_N}$$

③ Superposition theorem

a. make one source of voltage active at a time

b. take one voltage source ✓

c. And SHORT the other

d. Make two KVL equations and solve by ① in ckt equation.

e. same way, get I_1 ✓

f. Now do the same for another source, get I_2

$$I_{total} = I_1 + I_2 \checkmark$$

④ Source Transformation

UNIT-03-MAGNETIC CIRCUITS.

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String

LAWS OF MAGNETISM.

1) Like poles repel and unlike poles attract eio.

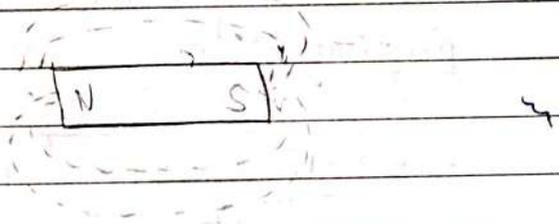
2) The force exerted by one pole on another pole is
∝ product of pole strength
Inversely proportional to square of distance b/w them.

$$F \propto \frac{m_1 m_2}{d^2} \quad F = \frac{k m_1 m_2}{d^2}$$

k = magnetic properties of medium.

*MAGNETIC CIRCUIT

Region around the magnet upto where its poles have force of attraction represented by imaginary lines of force around the magnet.



① Magnetic flux (ϕ) ✓

• The total magnetic lines of force in a magnetic field

• unit - weber

② Magnetic flux density (B) ✓

• magnetic flux passing through unit area of cross section.

$$B = \frac{\phi}{a} \quad \text{wb/m}^2 \text{ or Tesla}$$

③ Magnetising force / field strength (H)

• force which decides the strength of the magnetic field.

• gives the [quantitative measure] of the strength / weakness of magnetic material.

$$H = \frac{NI}{L} \quad \text{A/metre}$$

④ Permeability (μ) ✓

• Ability of a material to conduct flux.

• It is the ease with which the magnetic material allows flux to be set up through it.

$$\mu = \frac{B}{H} \quad \text{Henry/metre}$$

⑤ Absolute permeability (μ_0)

• The ratio of B to H for a magnet placed in [other than μ free] space or air or vacuum.

$$\mu_0 = \frac{B}{H}$$

⑥ Permeability of free space

• Ratio of B to H for a magnet placed in [free space] or air or vacuum.

$$\mu_0 = 4\pi \times 10^{-7} = 12.56$$

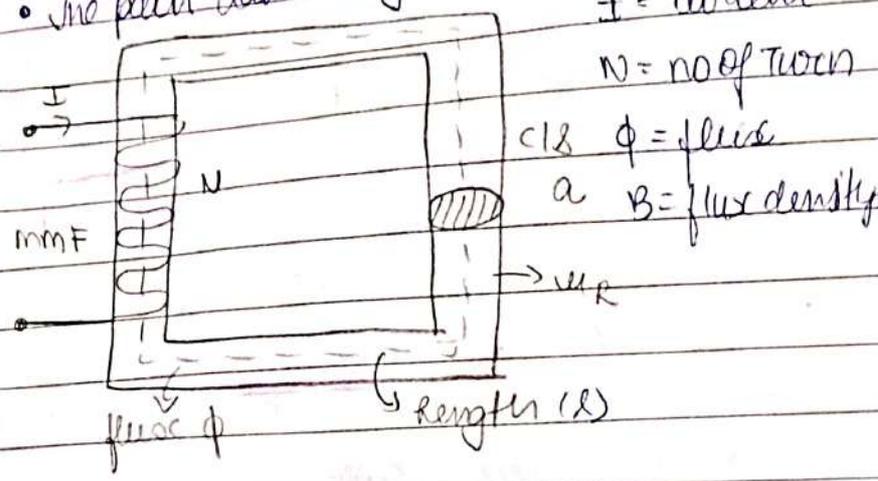
⑦ Relative permeability ✓

• permeability relative to the permeability of free space or air.

$$\mu_r = \frac{\mu}{\mu_0}$$

MAGNETIC CIRCUIT

The path taken by the magnetic flux



$I = \text{current}$
 $N = \text{no of turns}$
 $\phi = \text{flux}$
 $B = \text{flux density}$

$\mu = \text{Absolute permeability}$
 $\mu_{rel} = \text{Relative permeability}$
 $l = \text{mean length of flux path mels}$

Total flux in coil $\phi = B \cdot a$
 $= \mu H a$

$$= \frac{\mu \cdot NI a}{l}$$

$$\frac{\text{mmf}}{\text{Reluctance}} = \frac{NI}{\left(\frac{l}{\mu a}\right)}$$

• mmf = magnetomotive force (NI)
 • Reluctance = opposition to the magnetic flux

② $S = \frac{l}{\mu a}$ (analogous to $R = \rho \frac{l}{a}$)

• mmf - emf
 • reluctance - resistance
 • current and flux } Analogous terms.

ELECTROMAGNETIC INDUCTION

When conductor is moved in magnetic field, an emf is induced in the conductor.

The phenomenon of cutting the flux lines by the conductor to get the induced emf in it - EMI

- Requirements
 - conductor / coil
 - Magnetic field
 - Relative motion
 - blow coil and magnet

FARADAY'S LAW

① 1st law
 • whenever magnetic lines of force linking with the coils changes, and emf is induced

② 2nd law
 • Magnitude of induced emf is proportional to the rate of change of flux linkage (NΦ)
 $e = \frac{N\phi_2 - N\phi_1}{t} = \frac{Nd\phi}{dt}$

LENZ LAW

• But Lenz said that the direction of this emf is such that it will oppose the cause which produces it.

→ $e = -N \frac{d\phi}{dt}$ -ve sign at Lenz law

TYPES OF EMF

- ① Dynamically induced emf
 - Induced by physically moving either coil or magnetic field.
 - Let B = flux density
 - l = length
 - v = velocity.

Let conductor be moved through distance dn in time dt then.

Area swept by conductor: $a = l \cdot dn$.

Flux cut by conductor = $B \cdot l \cdot dn$.

$$e = \frac{d\phi}{dt} \cdot B \cdot l \cdot dn = Blv \text{ volt}$$

$$e = blv \sin \theta$$

② Self induced emf

- Induced in one coil because of rate of change of its own flux linked with it.
- $e = -N \frac{d\phi}{dt}$ multiply and divide by I

$$e = -N \frac{d\phi}{dt} \times \frac{I}{I}$$

If permeability of material remains constant the ϕ/I ratio is constant.

SELF INDUCTANCE

$$e = \frac{N\phi}{I} \frac{dI}{dt} \quad | \quad L = \frac{N^2}{S}$$

where $\frac{N\phi}{I} \rightarrow$ coefficient of self induction or self inductance $[e = -L \frac{di}{dt}]$

- unit of self inductance - Henry
- Flux linkages per ampere or property of coil to oppose change in current.
- $L = \frac{N\phi}{I} = \frac{N^2}{S} \because \phi = \frac{NI}{S}$

FACTORS AFFECTING self Inductance

- Directly proportional to square of number of turns.
- Magnetic properties of material
- $\propto \frac{1}{l}$
- $e \propto N^2$, $e \propto \frac{1}{l_{\text{core}}}$
- $e \propto \text{Area}$, $e \propto \mu$
- Also varies with current

MUTUAL INDUCTANCE

emf induced in a coil due to change in flux of other coil linked with it.

$$e_2 = N_2 \frac{d\phi_1}{dt}$$

$$= N_2 \frac{d(\mu_1 \times \frac{I_1}{l})}{dt}$$

$$= \frac{N_2 \phi_1}{I_1} \times \frac{dI_1}{dt} = M \frac{dI_1}{dt} \quad \text{Cmlin}$$

where M is called the coefficient of mutual inductance

• defined as the ability of one coil to induce an emf in the nearby coil by induction when the current in first coil changes

X X

* MUTUAL INDUCTANCE coupling constant

• Two coils having N_1 and N_2 turns carrying I_1 and I_2 Amp produces

$$L_1 = \frac{N_1 \Phi_1}{I_1} \quad \text{and} \quad L_2 = \frac{N_2 \Phi_2}{I_2}$$

• Let k_1 fraction of flux Φ_1 links coil 2.

$$M = \frac{N_2 k_1 \Phi_1}{I_1} \quad k_1 \leq 1 \rightarrow \textcircled{1}$$

• Let k_2 fraction of flux Φ_2 links coil 1.

$$M = \frac{N_1 k_2 \Phi_2}{I_2} \quad k_2 \leq 1 \rightarrow \textcircled{2}$$

Multiply $\textcircled{1}$ & $\textcircled{2}$

$$M^2 = \frac{k_2 N_2^2 \Phi_2}{I_2} \times \frac{k_1 N_1 \Phi_1}{I_1}$$

$$= k_1 k_2 \frac{N_2^2 \Phi_2}{I_2} \frac{N_1 \Phi_1}{I_1} = k_1 k_2 L_1 L_2$$

$$\therefore M = \sqrt{k_1 k_2} \sqrt{L_1 L_2} = k \sqrt{L_1 L_2}$$

• where k is called as the coefficient of coupling = $\frac{M}{\sqrt{L_1 L_2}}$

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

• Defined as the ratio of actual mutual inductance present between two coils to its maximum possible value.

- if $k = 1$ - tightly coupled
- if $k < 1$ - loosely coupled

* ENERGY STORED IN Magnetic field

• If i increases by di ampere in time dt sec then induced in the coil is

$$\rightarrow e = -L \frac{di}{dt} \text{ volt}$$

• The emf opposes the current & energy drawn from source to overcome this opposition is stored in MF.

• voltage to neutralise = $-e$.

• Energy absorbed in time dt

$$= \text{Power} \times \text{time}$$

$$= (e) i dt$$

$$= L \frac{di}{dt} \cdot i dt$$

$$= L i di \text{ joule}$$

Electric

DIFFERENCE

Magnetic

P
E
R
R

conductivity
current density
conductance
D
M

- 1) Here no current flows
- 2) Installation is not required
- 3) energy is not required

- 1) Flux sets up
- 2) NO installation
- 3) energy req only establish

Total energy absorbed when current reaches final value of I

$$= \int_0^I L i di = L \int_0^I i di = \frac{L}{2} (i^2)$$

$$= \frac{1}{2} LI^2 \text{ Joule.}$$

energy stored per unit volume

$$E = \frac{1}{2} \frac{N\phi}{I} \times I^2 \quad \because L = \frac{N\phi}{I}$$

$$= \frac{1}{2} N_0 B \cdot a I = \frac{1}{2} B \cdot a \cdot H \cdot l$$

($\phi = Ba$) ($NI = mmf$)

$$= \frac{1}{2} BH \text{ J/m}^3 \quad \because a \times l = \text{Volume}$$

$$\frac{1}{2} \mu H^2 \cdot \text{J/m}^3 \quad B = \mu H$$

$$\frac{1}{2} \frac{B^2}{\mu} \cdot \text{J/m}^3 \quad \because \mu = \frac{B}{H}$$

5) $I = \frac{mmf}{R}$

5) $\phi = \frac{mmf}{\text{Reluctance}}$

6) $R = \frac{l}{\mu a}$

6) $S = \frac{1}{\mu_0 \mu_r} \frac{A}{l}$

7) Voltage = IR

7) $mmf = \phi \cdot S$

8) Intensity

8) Magnetic Intensity

$E = \frac{V}{d}$

$H = \frac{NI}{l}$

9) Conductance

9) Permeance

$\frac{A}{R}$

$\frac{1}{S}$

10) Conductivity

10) Permeability

11) Current density

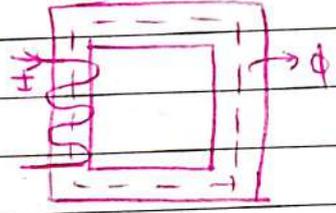
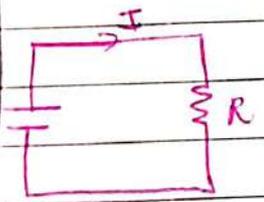
11) Flux density

$\frac{I}{a}$

$B = \frac{\phi}{a}$

ELECTRICAL circuit

Magnetic circuit



closed path for electrical current is called electrical circuit

1) closed path for flux is called magnetic circuit

current flows through it

2) Flux is set up in it

current flows due to emf

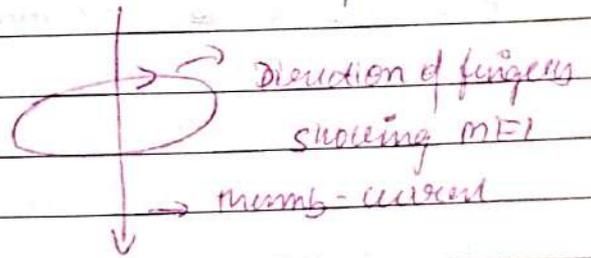
3) Flux is created due to mmf

Resistance opposes flow of current

4) Reluctance opposes the flux

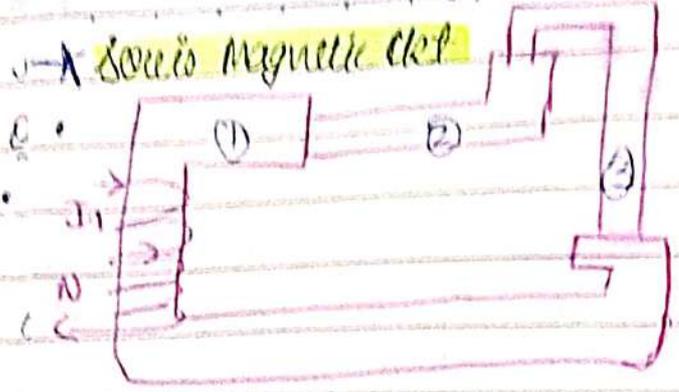
*** RIGHT HAND THUMB RULE ***

- Direction of induced emf is given by Fleming's right hand rule
- Hold the thumb and first two fingers of right hand mutually perpendicular to each other.
- If thumb shows direction of motion, and first finger shows the direction of magnetic field then middle finger points in the direction of induced emf



$\mu_r \mu_0 H = \frac{B}{\mu_0}$
 $\mu_r \mu_0 \frac{NI}{l} = \frac{B}{\mu_0}$
 $B = \mu_r \mu_0 NI$
 DERIVATIONS

$H = \frac{NI}{L}$
 $L = 2\pi r$
 $L = \text{distance of}$

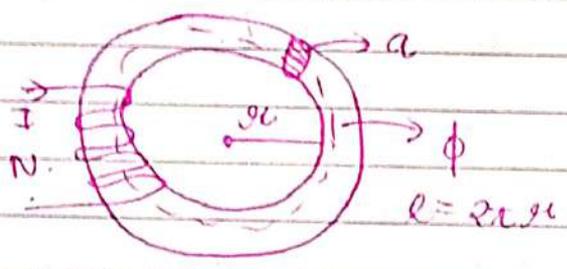


① MMF

\bullet $MMF = MMF_1 + MMF_2 + MMF_3$
 $= \phi S_1 + \phi S_2 + \phi S_3$
 $= \phi S_{\text{total}}$

② Reluctance $S_T = S_1 + S_2 + S_3$
 $S_1 = \frac{l_1}{\mu_0 \mu_r a_1}$

③ FLUX IN TOROID



② $B = \frac{\phi}{a}$ $\phi = B \cdot a$
 $\phi = \mu \cdot H \cdot a$
 $\phi = \mu_0 \mu_r \cdot H \cdot a$
 $\phi = \mu_0 \mu_r \cdot \frac{NI}{2\pi r} \cdot a$

$\phi = \frac{NI \times \text{MMF}}{\text{Reluctance}}$

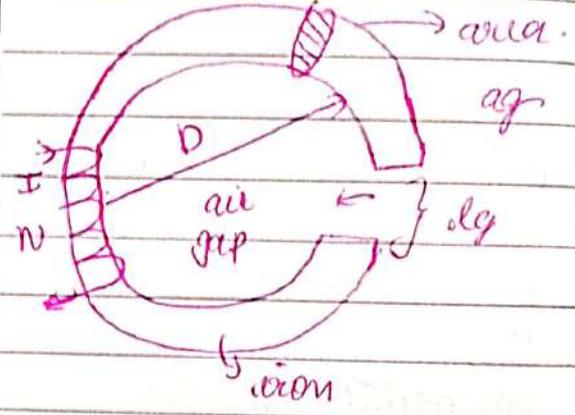
$\left(\frac{1}{\mu_r \mu_0} \right) \frac{l}{a}$
 Reluctance

$\phi = \frac{MMF}{\text{Reluctance}}$
 unit of flux is weber.

UNITS OF PARAMETERS

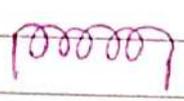
- Reluctance
 $S = \frac{MMF}{\phi}$, $S = \frac{l}{\mu_r \mu_0 a}$
- Flux - Weber
- $MMF = \phi \cdot S$, NI , (At)
- B - flux density = Tesla
- Inductance - Henry
- Magnetic strength (H) = $\frac{At}{l}$

IRON RING



$a_g = a_i$
 $B_g = B_i$
 $S_i = \frac{1}{\mu_0 \mu_r} \times \frac{l_i}{a_i}$
 $S_g = \frac{1}{\mu_0 \mu_r} \times \frac{l_g}{a_g}$
 $S_T = S_i + S_g$
 $l_i = \pi D$

SOLENOID



$H = \frac{NI}{L}$
 B_1
 $\mu_0 \mu_r$

Iron conductor



$H = \frac{NI}{L}$
 B_2
 $\mu_r \mu_0$

$B_1 < B_2$

DERIVATION OF COEFFICIENT OF COUPLING

equations for e1

e1 = -N1 dphi1 / dt

e1 = -L1 di1 / dt

e1 = -M di2 / dt

where L1 = N1 phi1 / I1

M1 = N1 k2 phi2 / I2 -> (1)

equations for e2

e2 = -N2 dphi2 / dt

e2 = -L2 di2 / dt

e2 = -M di1 / dt

where M2 = N2 k1 phi1 / I1 -> (2)

multiply (1) x (2)

M1 x M2 = (N1 k2 phi2 / I2) x (N2 k1 phi1 / I1)

M^2 = k1 k2 N1 N2 phi1 phi2 / I1 I2

M^2 = k1 k2 N1 phi1 / I1 x N2 phi2 / I2

M^2 = k1 k2 L1 L2

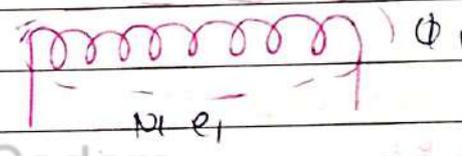
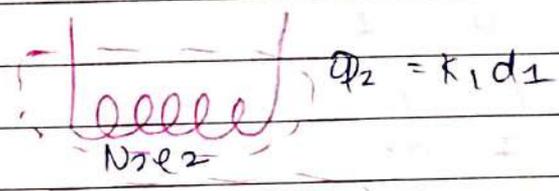
k1 = k2 = k

M^2 = k^2 L1 L2

M = k sqrt(L1 L2)

k = M / sqrt(L1 L2), k = 1 max value of mutual inductance

(m) max = sqrt(L1 L2)



SharkCoders

L = N phi / I = (N x NI) / S = N^2 / S

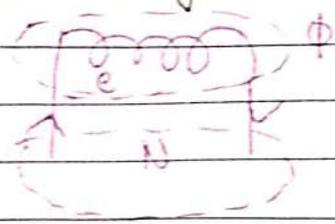
phi = NI / S

L = N^2 / S

M = (N1 phi2 k2) / I2 = (N2 k1 phi1) / I1

M = (N1 N2) / S

*** Electromagnetic INDUCTION.**



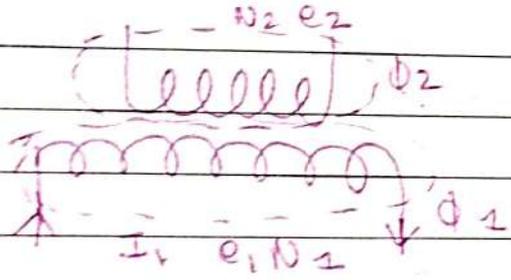
$e = - \frac{N d\Phi}{dt} = e = - \frac{N d\Phi}{dt}$

• Multiply & divide with I.

$e = - \frac{N d\Phi \times I}{dt \times I}$

$e = - \frac{N\Phi}{I} \frac{di}{dt}$

*** MUTUAL INDUCTANCE**



SharkCoders

$e_1 = - \frac{N_1 d\Phi_1}{dt} = - \frac{N_1 d\Phi_1}{dt} \times \frac{I_1}{I_1}$
 $= - \frac{N_1 d\Phi_1}{dt} \times \frac{I_1}{I_1} = - \frac{N_1 \Phi_1}{I_1} \times \frac{di}{dt}$
 $= e = - (L_1) \times \frac{di_1}{dt}$

$e_2 = - \frac{N_2 d\Phi_2}{dt} = - \frac{N_2 d\Phi_2}{dt} \times \frac{I_2}{I_2}$

$e_2 = - \frac{N_2 \Phi_2}{I_1} \times \frac{di_2}{dt}$

$e = - N_2 \frac{d\Phi_1}{dt}$

$e_2 = - \frac{N_2 k_1 \Phi_1}{I_1} \frac{di_1}{dt}$

↳ multiply divide

$e_2 = \left(\frac{N_2 k_1 \Phi_1}{I_1} \right) \frac{di_1}{dt}$

$e_2 = M \frac{di_1}{dt}$

where $M = \frac{N_2 k_1 \Phi_1}{I_1}$

$M = \frac{N_1 \Phi_1}{I_1}$

$\mu = \frac{N_1 N_2}{S} \quad \mu = \frac{k N_1 N_2}{S}$

*** FORMULAE.**

① when l_i, l_g, μ, B given and calculate mmf

$\text{mmf} = H_i l_i + H_g l_g$

$= B \times l_i + B \times l_g$

where where

use $\text{mmf} = B \left(\frac{l_i}{\mu_{core}} + \frac{l_g}{\mu_0} \right)$

in ratio form.

or use $\text{mmf} = NI$ To find

1) In N_1, N_2 different coils,
 $\mu_1, \mu_2, \mu_3, \mu_4$, given and
flux is asked.

Find S_1 and S_2 .

then $\phi = \frac{\text{MMF}}{S_1 + S_2}$

$S_1 + S_2$

* RIGHT HAND THUMB RULE

Hold current carrying conductor
in the right hand such that
thumb pointing in the
direction of current and parallel
to the conductor then tips
of curled fingers point in the
direction of MF.

Shark Coders

* MMF

• unit = $NI = AT$

• Quantity appearing in the equation
for the magnetic flux in a
magnetic circuit.

• Force produced by current
through a coil of wire that
gives rise to MF.

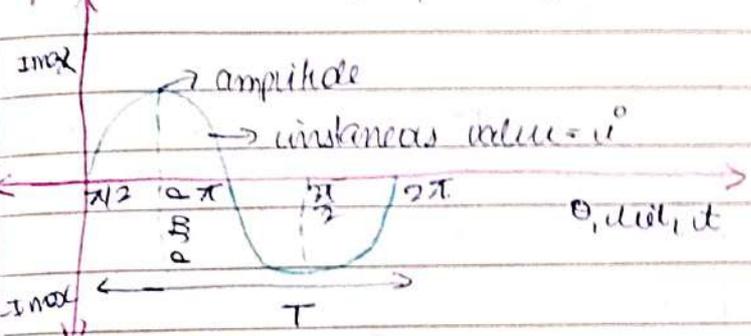
• Product of $\phi \times$ Reluctance.

BEE - UNIT - 01 - AC FUNDAMENTALS

DEFINITIONS

Waveform

Algebraic form / graphical representation of an alternating quantity.



$i = I_{max} \sin(\omega t)$ Amp

Cycle

set of all possible instantaneous values obtained by that cycle.

Instantaneous value

value of that quantity at a given instant.

Time period (T), $T = 2\pi/\omega$

Time required to complete one cycle (in seconds).

Frequency $f = \frac{1}{T}$

Number of cycles completed in per second, Hertz.

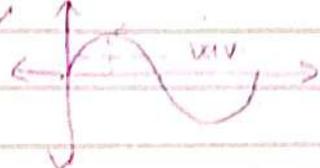
Amplitude (A)

Maximum value obtained by alternating quantity.

$m = A \sin \omega t + \phi$
 $\rightarrow A$ is amplitude.

1) Average value

If we connect the AC sine wave to a sine wave through rectifier, then the average value of the DC is known as average value.



Average value of $I_{av} = 0.637 I_{max}$ current

2) Peak Factor

It is the ratio b/w maximum value and RMS value of an alternating wave.

$PF = \frac{\text{Max Value}}{\text{RMS}}$

Also known as crest / Amplitude factor.

$PF = \frac{I_{max}}{0.707 I_{max}} = 1.414$

3) Form factor

Ratio b/w RMS value & average value of an alternating quantity.

Form factor = $\frac{\text{RMS}}{\text{Average}}$

$\frac{0.707}{0.637} = 1.11$

10) RMS value - Square root of the mean of the squares of the instantaneous voltages over a cycle.

*** AVERAGE VALUE DERIVATION.**

$I_{avg} = \frac{1}{\pi} \int_0^{\pi} i^2 dt$

$\frac{1}{\pi} \int_0^{\pi} I_{max} \sin \omega t dt$

$I_{max} [-\cos \omega t]_0^{\pi}$

$I_{max} (-\cos \pi + \cos 0)$

$I_{max} - (-1) + (1)$

$I_{avg} = \frac{2 I_{max}}{\pi}$

$I_{avg} = \frac{2}{\pi} \times I_{max}$

$I_{avg} = 0.636 I_{max}$

$I_{rms} = \text{RMS of } i = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} i^2 dt}$

$= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} I_m^2 \sin^2 \omega t dt}$

$= \sqrt{\frac{I_m^2}{2\pi} \int_0^{2\pi} (1 - \cos 2\omega t) dt}$

$= \sqrt{\frac{I_m^2}{4\pi} [t - \frac{\sin 2\omega t}{2\omega}]_0^{2\pi}}$

$= \sqrt{\frac{I_m^2}{4\pi} [2\pi - \frac{\sin 4\pi}{2\omega} - 0 - \frac{\sin 0}{2\omega}]}$

$= \sqrt{\frac{I_m^2}{4\pi} [2\pi - 0 - 0 - 0]}$

$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707 I_{max}$

*** RMS VALUE DERIVATION.**

$I_{eff} = \sqrt{\frac{i_1^2 + i_2^2 + \dots + i_n^2}{n}}$

→ Root mean of square of all instantaneous values in a cycle.

$i = I_{max} \sin \omega t$ Amp

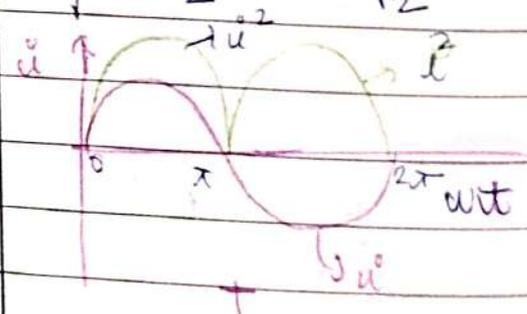
squared $i^2 = i^2 = I_{max}^2 \sin^2 \omega t$

$\Sigma \text{ of } i^2 = \int_0^{2\pi} i^2 dt$

Mean of $i^2 = \frac{1}{2\pi} \int_0^{2\pi} i^2 dt$

$= \sqrt{\frac{1}{4\pi} \int_0^{2\pi} I_m^2 \sin^2 \omega t dt}$

$= \sqrt{\frac{I_{max}^2}{2} \times \frac{1}{2} \times 2\pi} = \frac{I_{max}}{\sqrt{2}} = 0.707 I_{max}$



FORMULAE

$I_{avg} = 0.636 I_{max}$

$I_{RMS} = 0.707 \times I_{max}$

$I_{RMS} = \frac{I_{max}}{\sqrt{2}}$

milli seconds = 10^{-3}

If $f = 50$ the $\omega = 314$.

After what ^{time} will the current becomes $86.6 A$

$i = I_{max} \sin \omega t$

current is $86.6 A$, find t ?

$i = 28 \sin 314 t$

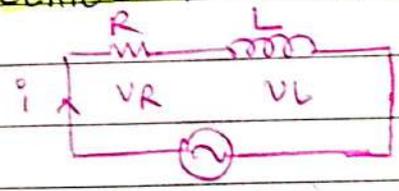
in calculator $pld \rightarrow$ RADIAN

$28 \times \sin \times 314 \times t$

$\frac{14}{28} = \sin \times \omega t$

so $\sin^{-1} \left(\frac{14}{28} \right) = \omega t$

SERIES R-L CIRCUIT



$v = v_m \sin \omega t$

$\bar{v} = \bar{v}_R + \bar{v}_L$

$= IR + I X_L$

$I (R + X_L)$

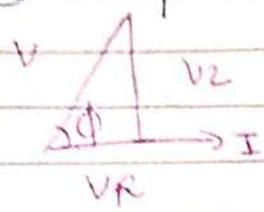
$v = I Z$

where Z is impedance (opposition to AC current)

Applied voltage = $v = v_m \sin \omega t$

current = $i = I_m \sin \omega t (-\theta)$

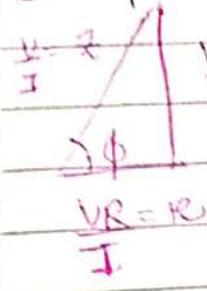
① Voltage Triangle



$V_L = V \sin \phi$

$V_R = V \cos \phi$

② Impedance Triangle (Dividing by I)



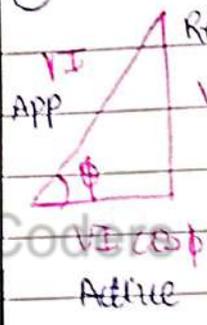
$\frac{V_L}{I} = X_L$

$X_L = Z \sin \phi$

$R = Z \cos \phi$

$\therefore Z \cos \phi = R + j X_L$

③ Power Triangle (Multiply by I)



Active Power

$\rightarrow VI \cos \phi$ VAR

Reactive Power

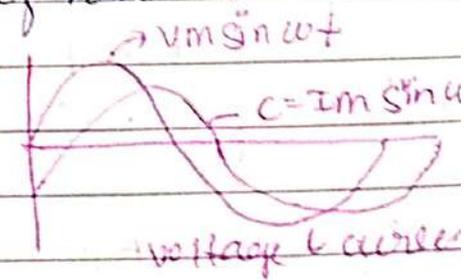
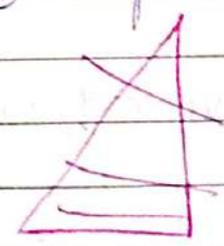
$\rightarrow VI \sin \phi$ VAR

Apparent power $VI = VA$

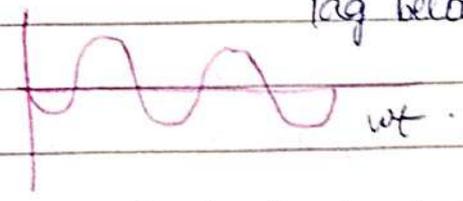
where $\cos \phi$ is the power factor

\rightarrow Ratio of $\frac{\text{active power}}{\text{Apparent}}$

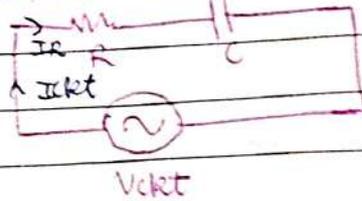
④ waveform of Power



lag behind



SERIES RC CIRCUIT

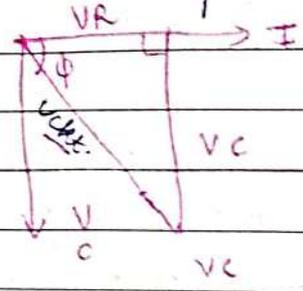


applied voltage
 $V = V_m \sin \omega t$
 current circuit
 $i = I_m \sin(\omega t + \phi)$

$$= V_m \sin \omega t \sin(\omega t + \phi)$$

$$= \frac{V_m I_m \cos \phi}{2} - \frac{V_m I_m \cos(2\omega t + \phi)}{2}$$

① Phasor diagram

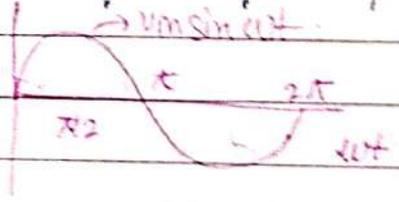


$V_C = V \sin \phi$
 $V_R = V \cos \phi$

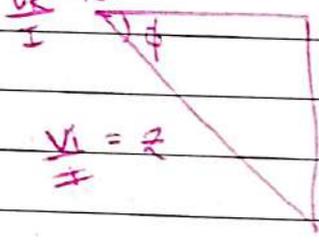
$$= \frac{V_m I_m \cos \phi}{2} - \frac{V_m I_m \cos(2\omega t + \phi)}{2}$$

$V = \bar{V}_R + \bar{V}_C$
 $I R + I X_C$
 $I (R + X_C)$
 $V = I Z$

④ waveform of voltage

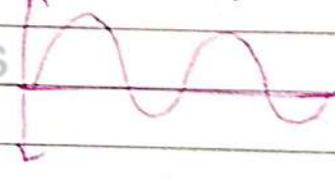


② Impedance triangle (Divide by I)



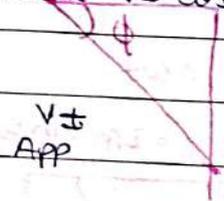
$\frac{V_C}{I} = X_C$ $X_C = Z \sin \phi$
 $R = Z \cos \phi$
 $\therefore Z \angle -\phi$ or
 $Z = R - jX_C$

⑤ waveform of power



$v = V_m \sin \omega t$
 $i = I_m \sin(\omega t + \phi)$
 $P = VI \cos \phi$

③ Power Triangle (Multiply by I)



Active = $VI \cos \phi$ watts
 Reactive = $VI \sin \phi$ VAR
 Apparent = VI VA

Instantaneous power is given by

$p = v_c i = V_m \sin \omega t \cdot I_m \sin(\omega t + \phi)$

* SERIES R-L-C CIRCUIT



$V = V_m \sin \omega t$

$$v = v_R + v_L + v_C$$

$$= IR + IX_L + IX_C$$

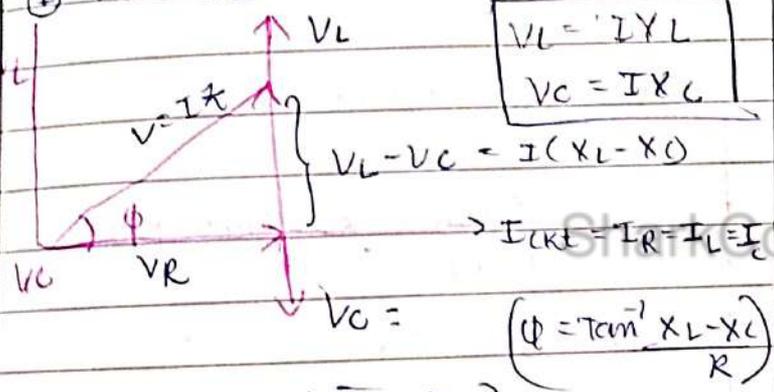
$$= I(\bar{R} + \bar{X}_L + \bar{X}_C)$$

$$V = IZ$$

where Z is impedance

X_L and X_C are in phase opposition so consider 3 conditions:

① $X_L > X_C \therefore V_L > V_C$



$$\bar{V} = \bar{V}_R + (\bar{V}_L - \bar{V}_C)$$

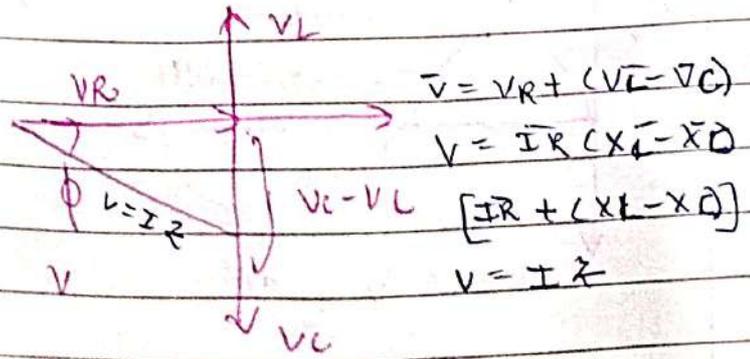
$$= I\bar{R} + (IX_L - IX_C)$$

$$= I[R + (X_L - X_C)]$$

$$\bar{V} = I\bar{Z}$$

where $Z = \sqrt{R^2 + (X_L - X_C)^2}$

② $X_L < X_C \therefore V_L < V_C$



$$\bar{V} = \bar{V}_R + (\bar{V}_C - \bar{V}_L)$$

$$= I\bar{R} + (IX_C - IX_L)$$

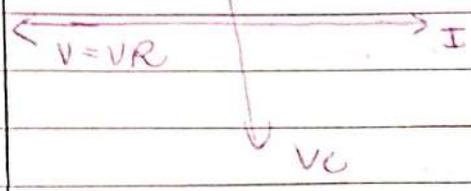
$$= I[R + (X_C - X_L)]$$

$$\bar{V} = I\bar{Z}$$

where $Z = \sqrt{R^2 + (X_C - X_L)^2}$

$\phi_{\text{lag}} = \tan^{-1} \left(\frac{X_C - X_L}{R} \right)$

③ $X_L = X_C \therefore V_L = V_C$ hence cancel each other



$\therefore V = V_R$

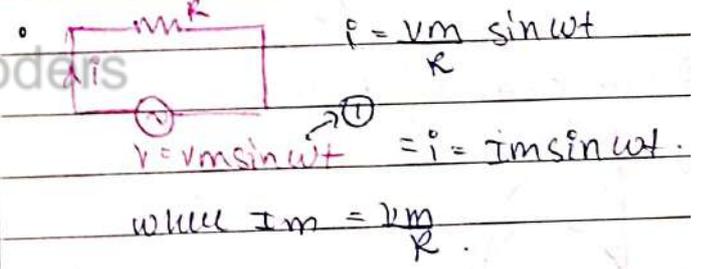
This is called series resonance

$I = I_R = I_L = I_C = I \cos \phi$

with \bar{Z} and $\bar{V} = + - + i_n$ all conditions

and with $V_L = IX_L, V_C = IX_C$

* AC FOR PURE RESISTANCE



where $I_m = \frac{V_m}{R}$

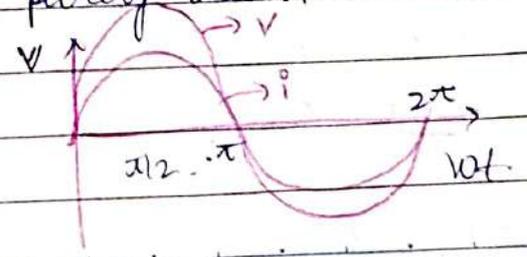
$i = \frac{v_m \sin \omega t}{R}$

$i = \frac{V_m}{R} (\sin \omega t)$

$i = I_{\text{max}} \sin \omega t \rightarrow \text{②}$

From ① and ②

V and I are in phase for purely resistive circuit.



① Phase



② Equation of Power
 $P = VI$

$$V_m \sin \omega t \cdot I_m \sin \omega t$$

$$V_m I_m \left(\frac{1 + \cos 2\omega t}{2} \right)$$

$$P_{inst} = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$$

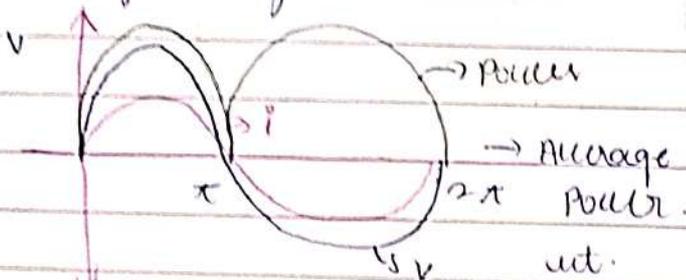
↳ Instantaneous power

• Average power =

$$P_{avg} = \frac{V_m I_m}{2} = 0$$

$$P_{avg} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = VI$$

③ waveform of Power



$$P = \frac{V_m I_m}{2} \cos^2 \omega t$$

AC TO PURE INDUCTANCE



self induced emf will always oppose

$v = v_m \sin \omega t$ applied voltage

As per Lenz law

$$v = -e$$

$$v = -e = -L \frac{di}{dt}$$

$$v_m \sin \omega t = L \frac{di}{dt}$$

$$di = \frac{v_m \sin \omega t}{L} dt$$

$$i = \int \frac{v_m \sin \omega t}{L} dt$$

Integrating both sides

$$i = \int \frac{v_m \sin \omega t}{L} dt$$

$$i = \frac{v_m}{L} \left(\frac{-\cos \omega t}{\omega} \right) dt$$

$$i = \frac{v_m}{\omega L} (-\cos \omega t)$$

$$i = \left(\frac{v_m}{\omega L} \right) \sin (\omega t - 90^\circ)$$

$$i = I_m \cos \sin (\omega t - 90^\circ) \rightarrow (1)$$

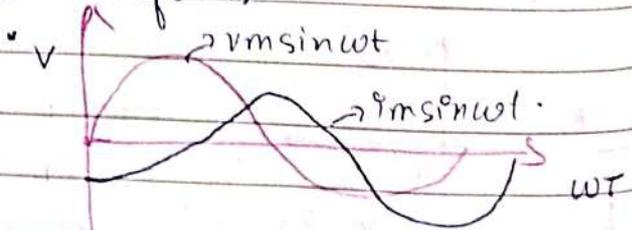
$$\text{where } I_m = \frac{V_m}{\omega L} = \frac{V_m}{X_L}$$

$$\frac{V_m}{X_L}$$

↳ X_L is Reactance

$$X_L = \frac{V}{I}$$

① waveform



I lags V by 90°

★ AC THROUGH PURE CAPACITANCE 06.

② Phase of RL circuit

$i = I_m \sin(\omega t - \frac{\pi}{2})$ where $I_m = \frac{V_m}{\omega L}$

③ Power

$P = VI$

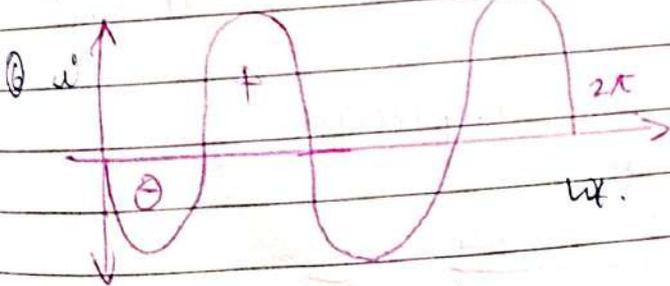
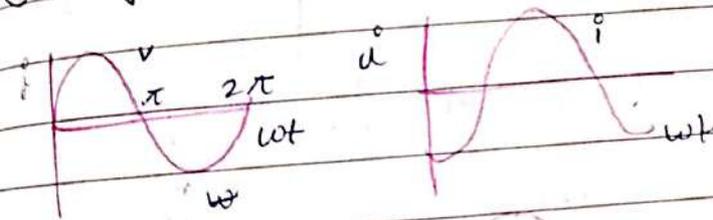
$P = v_m \sin \omega t \cdot I_m \sin(\omega t - 90^\circ)$
 $v_m I_m \sin \omega t (-\cos \omega t)$

$P_{\text{ins}} = -\frac{v_m I_m}{2} \sin 2\omega t$

Power in RL circuit = 0.

④ Voltage

⑤ Current

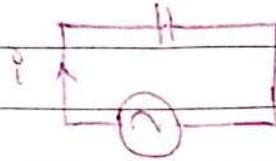


Power = 0 for 7 cycle.

$v = v_m \sin \omega t$

$i = I_m \sin(\omega t - \frac{\pi}{2})$

$P_{\text{ins}} = \frac{v_m I_m}{2} \sin(2\omega t)$



$Q = CV$

$V = v_m \sin \omega t$

$i = \frac{dq}{dt} = \frac{d(CV)}{dt}$

$i = C v_m \frac{d \sin \omega t}{dt}$

$= C v_m \cos \omega t \cdot \omega$

$i = \omega C v_m \sin(\omega t + 90^\circ)$

where $I_m = \omega C v_m$

$i = I_m \sin(\omega t + 90^\circ)$

$I_m = \frac{v_m}{\frac{1}{\omega C}}$

$X_C = \frac{1}{\omega C} \rightarrow$ capacitive reactance

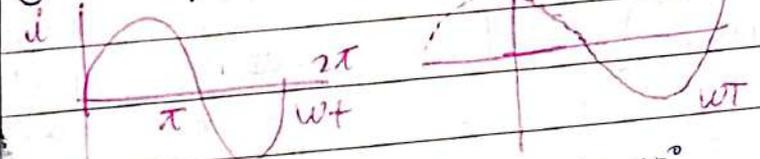
$I_m = \frac{v_m}{X_C}$

From ① and ②, I leads V by 90°

④ waveform of R-C circuit

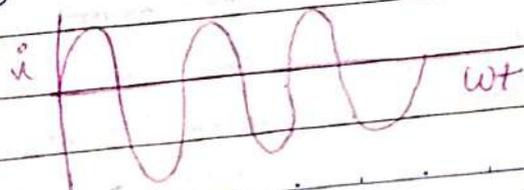
① Voltage

② Current



lead by 90° current.

③ Power.



FORMULA AND SUMMARY

① RLC

• $X_L = \omega L$

• $X_C = \frac{1}{\omega C}$

• $V = IR$

• $I_{ckt} = \frac{V_{ckt}}{Z_{ckt}}$

• $V = IZ$

• $\phi_{ckt} = \tan^{-1} \left(\frac{X_L - X_C}{R} \right)$
(degree)

• $P = VI \cos \phi$ (Active Power) watt

• $S = VI$ VA (Apparent)

• $Q = VI \sin \phi$ VAR (Reactive)

• $Z_{ckt} = \sqrt{R^2 + (X_L - X_C)^2}$
or $(X_C)^2$

• Show arrow \uparrow in phasor.

• Take rms value.

② Power Triangle in circuits.

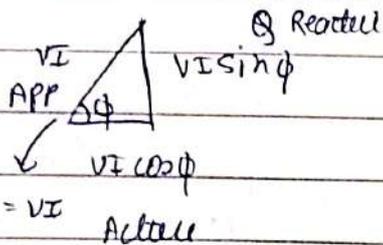
• BA - Base Active $\cos \phi$ (P)

• PR - Perpendicular Reactive $\sin \phi$ (Q)

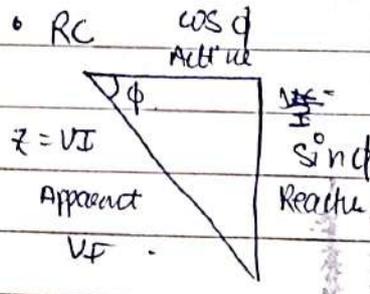
• HA - Hypotenuse Apparent $S = VI$

By multiplying I.

• RL



• RC



where $\cos \phi$ is power factor.
Active
Apparent

$X_C = Z \sin \phi$
 $R = Z \cos \phi$
 $VI \cos \phi$ watt
 $VI \sin \phi$ VAR
Apparent = VI Va.

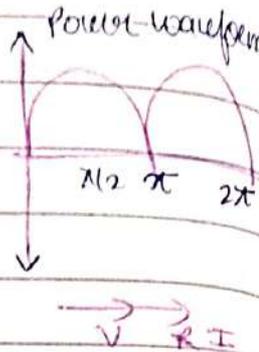
③ AC through

a. Pure Resistance.

• $P_{avg} = \frac{V_m I_m}{2} = 0$

$P_{avg} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = VI$

wave as $\frac{1 - \cos 2\omega t}{2}$



b. Pure Inductance

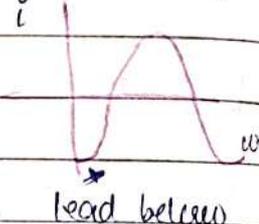
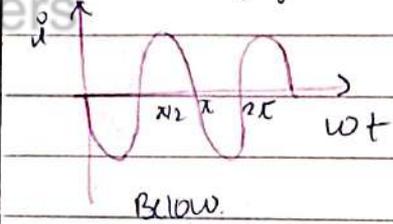
• $P_{avg} = 0$

• $P_{inst} = \frac{V_m I_m}{2} \sin 2\omega t$

• Find reactance $\frac{V_m}{I_m}$

$X_L \rightarrow$ Reactance

• Power waveform | • Current lag



c. Pure capacitance

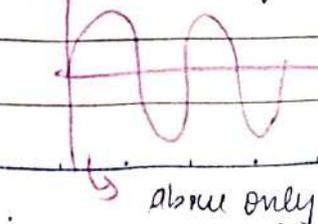
• By $q = CV$

• $I_m = \frac{V_m}{\frac{1}{\omega C}}$

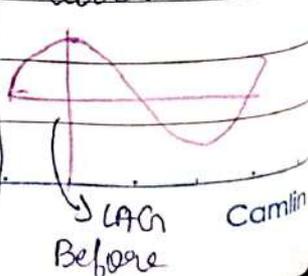
• $X_C = \frac{1}{\omega C}$

$\omega C \rightarrow$ capacitive Reactance

• Power waveform



• current



EFFECTIVE VALUE OF AC

Energy consumed by i_1 amp
 $\rightarrow i_1^2 R \frac{T}{n}$ joule

$\rightarrow i_2^2 R \cdot \frac{T}{n}$ joule

Energy consumed by i_n amp -
 $i_n^2 R \frac{T}{n}$ joule.

Total energy in a cycle

$$\left(i_1^2 \frac{RT}{n} + i_2^2 \frac{RT}{n} + \dots + i_n^2 \frac{RT}{n} \right)$$

$\rightarrow \textcircled{1}$

Energy given by direct current

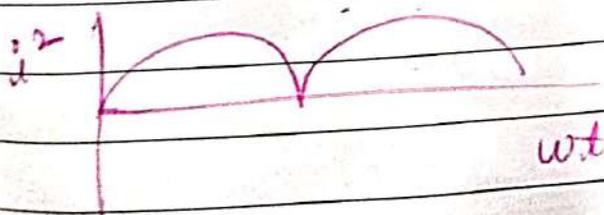
$\rightarrow I_{\text{eff}}^2 \times RT$ joule $\rightarrow \textcircled{2}$

$$I_{\text{eff}}^2 RT = \left[i_1^2 \frac{RT}{n} + i_2^2 \frac{RT}{n} + \dots + i_n^2 \frac{RT}{n} \right]$$

$$I_{\text{eff}}^2 RT = (i_1^2 + i_2^2 + \dots + i_n^2) \frac{RT}{n}$$

$$I_{\text{eff}} = \sqrt{\frac{i_1^2 + i_2^2 + \dots + i_n^2}{n}}$$

I_{eff} = Root mean of square of instantaneous values in a cycle.



UNIT-05-TRANSFORMERS & POLYPHASE CIRCUITS.

TRANSFORMER ✓

Static device which transfers power from one circuit to another at same frequency but at different voltage levels.

works on the principle of mutual induction

→ Two coils wound on the same magnetic core. The emf induced in one coil is due to the rate of change of flux in another coil.

Winding which is connected to the primary source of ^{energy} winding is called primary winding.

The winding which plays the secondary role in transfer of power or the winding to which the load is connected is called secondary winding.

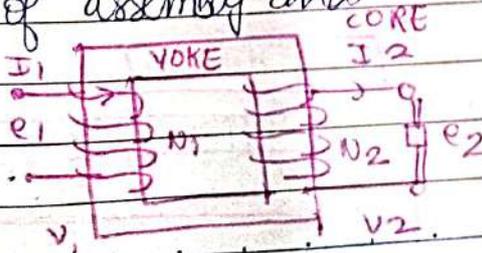
The part of magnetic core on which winding is placed is called limb.

The part of magnetic core which completes magnetic circuit through low reluctance path - yoke.

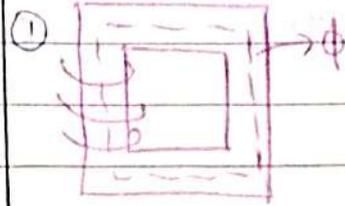
TYPES OF TRANSFORMER

on the basis of assembly and winding

- ① core type
- ② shell type



* CORE TYPE ✓



- Internal-core
- outer-winding

② Structure
• core is surrounded by winding

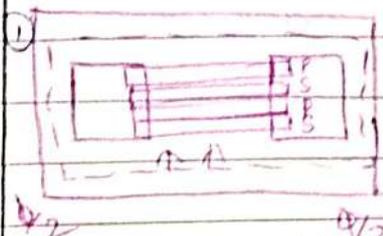
③ Rating
• High

④ Repair/Maintenance
• easy

⑤ Flux-straight path
⑥ windings
• cylindrical

⑦ cooling - Better

SHELL TYPE ✓



- Internal-winding
- outer-core

② Structure
• windings is surrounded by core

③ Rating
• Low

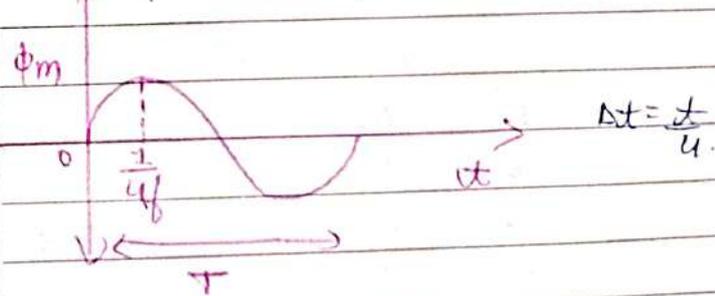
④ Repair/Maintenance
• Difficult

⑤ Flux-distributed
⑥ windings
• Sandwiched

⑦ cooling - Poor.

* EMF EQUATION OF TRANSFORMER

① waveform of flux.



• let $N_1, N_2, E_1, E_2, \Phi_m$ = max value of flux.

• Average induced emf is given by
= $\frac{d\Phi}{dt}$

$$= \frac{d\phi}{dt} \quad |E_{avg}| = \frac{N d\phi}{dt} = \frac{N \phi_{max} \cdot \omega}{T} \quad (T=4)$$

$$(\Delta\phi = \phi_{max} \sin \omega t), \Delta t = \frac{T}{4}$$

$$= \frac{N \omega \phi_{max}}{T}$$

$$|E_{avg}| = 4 f \phi_{max} N$$

For sinusoidal wave - form factor is 1.11.

$$RMS \text{ value} = \frac{4.44}{4.44} \frac{E_{avg}}{E_{RMS}} = 1.11$$

$$E_{RMS} = E_{avg}$$

$$E_{RMS} = 1.11 \times 4 f \phi_{max} N$$

$$E_{RMS} = 4.44 f \phi_{max} N$$

$$\text{emf induced in primary} = 4.44 \phi_{max} f N_1$$

$$\text{emf induced in secondary} = 4.44 \phi_{max} f N_2$$

* TRANSFORMER RATIO ✓

$$P_1 = P_2$$

$$V_1 I_1 = V_2 I_2$$

$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2} = K$$

\downarrow voltage transformation ratio
 \downarrow Turn's Ratio
 \downarrow current transformation ratio

$K > 1$ - step up transformer

$K < 1$ - step down transformer

$K = 1$ - isolation transformer

* TYPES OF LOSSES IN TRANSFORMER

1) iron losses / core / constant losses

a. Hysteresis loss

When MF is set up the molecules align in the direction of field, but if the field is changing/alternating they are reluctant to change direction, so energy is spent in changing the direction is called hysteresis loss.

$$W_h = k_h \cdot B_m^{1.6} \cdot f \cdot V \text{ watt}$$

k_h - hysteresis constant
 f - frequency
 V - volume of core

b. eddy current loss ✓

Due to linking of alternating to core, emf gets induced in the core giving rise to circulating currents in it.

These currents - eddy current.

every path of circulating current produces $I^2 R$ losses, called as eddy current losses.

$$W_e = k_e \cdot B_m^2 \cdot f^2 \cdot t^2 \cdot V \text{ watt}$$

B_m - max flux density

t - thickness of lamina

Flux density remains constant and supply frequency 100, therefore called as constant losses.

Copper Loss

• occurs in primary & secondary windings due to resistance of primary and secondary windings. $I^2 R$ losses

• Total Copper Losses - $I_1^2 R_1 + I_2^2 R_2$
 primary secondary
 $P_i + P_{cu}$

VOLTAGE REGULATION

• Change in secondary voltage expressed as fraction of no load secondary voltage when primary voltage remains constant.

• Regulation: $\frac{V_2(0) - V_2}{V_2(0)}$ at $V_1 = \text{const}$

EFFICIENCY OF TRANSFORMER

• efficiency $\eta = \frac{n \times V_A \times P_f}{(n \times V_A \times P_f) + \text{losses}}$

① $\text{Output} = n \times V_A \text{ Rating} \times P_f$
 $n = 1$ full load
 $V_A = \text{Rating} / \text{kVA} \rightarrow V_A \text{ V} \times 10^3$
 $P_f = \text{power factor value}$

Phases - $P_i = \text{constant (given)}$
 $(P_{cu})_{FL} = \text{V given}$
 $(P_{cu})_{\text{given load}} = (P_{cu})_{FL} \times n^2$
 $(P_{cu})_{GL} = \text{V}$

Total loss - $P_i + (P_{cu})_{\text{given}}$

• $n =$ loading fraction
 $P_i \text{ V, } P_{cu} \text{ FL V}$

Solving efficiency numericals

• Convert kVA into kVA $\rightarrow V_A \times 10^3$
 $S = \text{Apparent Power} = V_A R$

POLYPHASE AC CIRCUIT

① Symmetrical system (3ph)
 • (Balanced system) - is the system in which all the three voltages of same frequency are equal in magnitude and are displaced from each other by equal 120° angle.

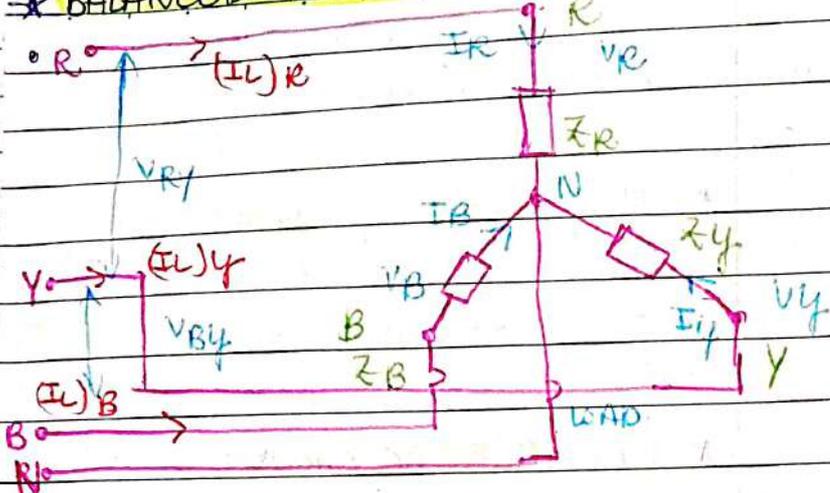
② Symmetrical load / Balanced load
 • Load in which each phase is equal in magnitude and identified in nature.

③ Phase sequence
 • The sequence in which three voltages achieve their positive max values.

RATIO NUMERICAL

• Tx Ratio = $\frac{N_2}{N_1} = \frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2}$
 • $\frac{E_1}{N_1} = \frac{E_2}{N_2}$ Voltage per Turn

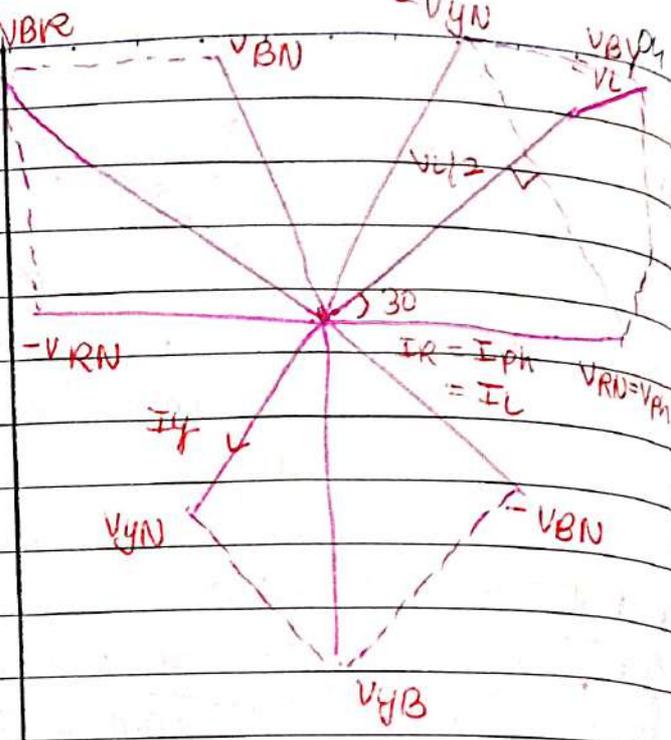
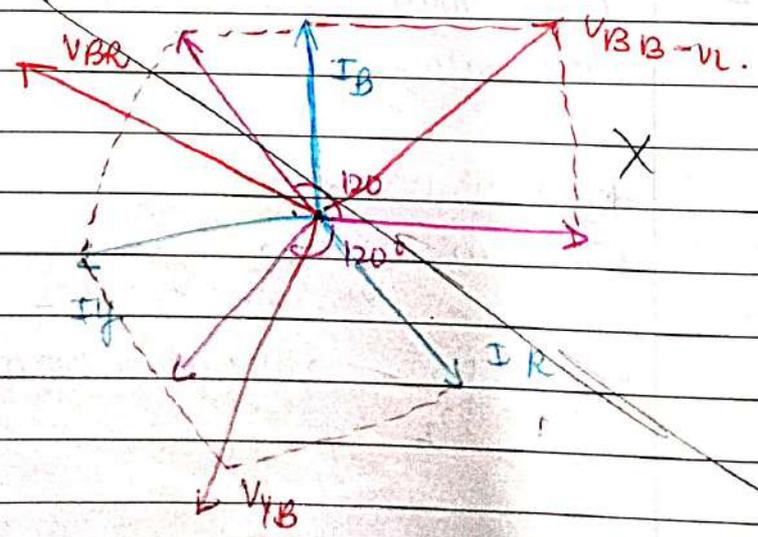
BALANCED STAR CONNECTED LOAD



- line voltage $V_L = V_{RY} = V_{YB} = V_{BR}$
- line currents $I_L = I_R = I_Y = I_B$
- Phase voltages $= V_{ph} = V_{RN} = V_{Yn} = V_{BN}$
- Phase current $= I_{ph} = I_R = I_Y = I_B$

Since the phase arm and line conductor form series circuit, hence $I_L = I_{ph}$ but

$V_L = V_{RY} = V_{RN} + V_{NY} = V_{RN} + (-V_{YN})$
 $V_{BR} = V_{BN} + V_{NR} = V_{BN} + (-V_{RN})$
 hence



$\cos 30^\circ = \frac{V_L/2}{V_{ph}}$

$\frac{\sqrt{3}}{2} = \frac{V_L}{2V_{ph}}$

$\therefore V_L = \sqrt{3} V_{ph}$
 $I_L = I_{ph}$

3 phase Active power = $3 V_{ph} I_{ph} \cos \phi = \sqrt{3} V_L I_L \cos \phi$ watt

Reactive power $3 V_{ph} I_{ph} \sin \phi = \sqrt{3} V_L I_L \sin \phi$ VAR

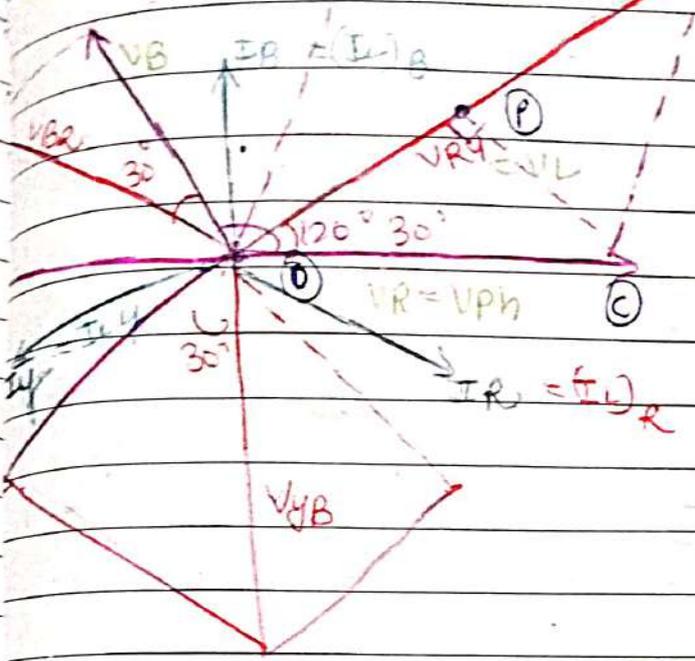
Apparent Power $3 V_{ph} I_{ph} = \sqrt{3} V_L I_L$ VAR

PHASOR-STAR-equations

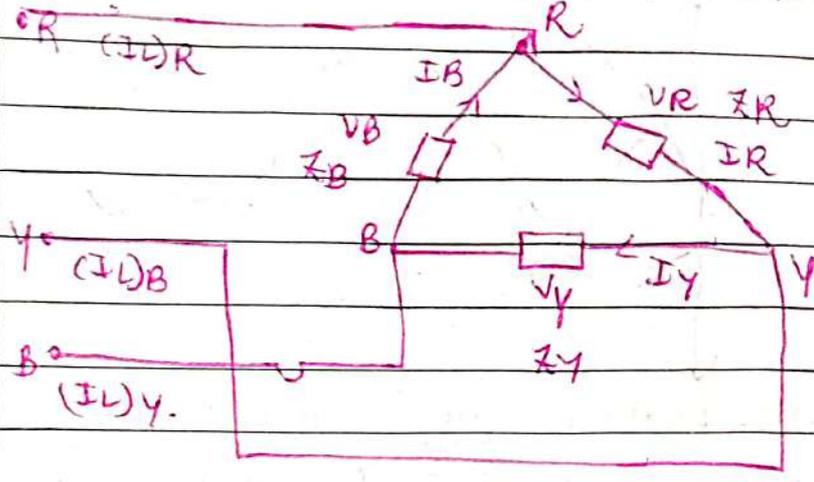
$V_{RY} = V_R - V_Y$
 $V_{YB} = V_Y - V_B$
 $V_{BR} = V_B - V_R$

$I_R, I_Y, I_B \rightarrow$ current

STAR-PHASOR



DELTA CONNECTION



STAR - Relation b/w VL & Vph

In parallelogram OACB

$l(OC) = V_R = V_{ph}$
 $l(OB) = V_{RY} = V_L$
 $l(OP) = V_L/2$

In ΔOPC

$\cos 30^\circ = \frac{l(OP)}{l(OC)} = \frac{V_L/2}{V_{ph}}$

$\frac{\sqrt{3}}{2} = \frac{V_L/2}{V_{ph}}$

$\frac{V_L}{2} = \frac{\sqrt{3}}{2} V_{ph}$

$V_L = \sqrt{3} V_{ph}$

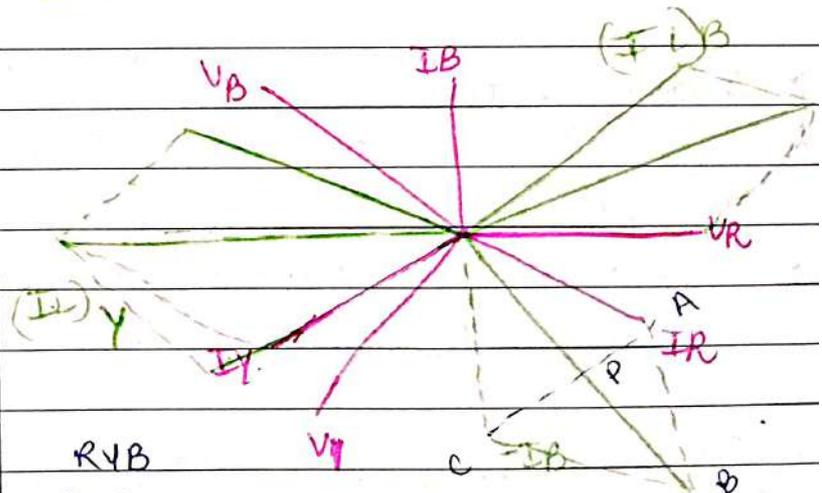
- $V_L = V_{ph}$ ✓ Delta ✓
- $l(OA) = I_R = I_{ph}$
- $l(OB) = (I_L)_R = I_L$
- $\Delta OAP, \angle P = 90^\circ$
- $l(OP) = l(PA) = \frac{1}{2} I_L$

$\cos 30^\circ = \frac{l(OP)}{l(OA)} = \frac{1/2 I_L}{I_{ph}}$

$\frac{\sqrt{3}}{2} I_{ph} = \frac{I_L}{2}$

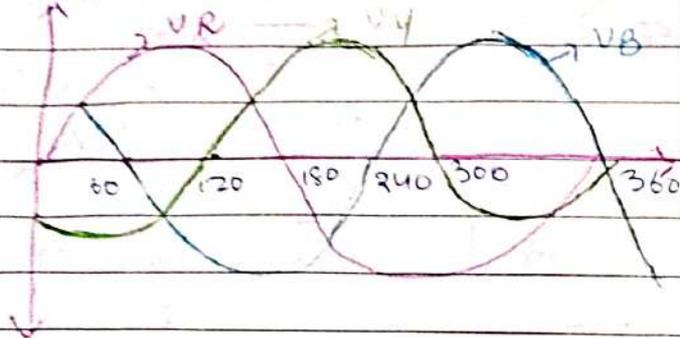
$I_L = \sqrt{3} I_{ph}$

DELTA PHASOR



- $(I_L)_R = I_R - I_B$
- $(I_L)_Y = I_Y - I_R$
- $(I_L)_B = I_B - I_Y$

WAVEFORM OF 3PHASE AC CIRCUIT



STAR

DELTA

- | | |
|--|---|
| <p>① 4 wire connection (4th wire - optional)</p> <p>② Two types
→ 3 Phase 4 wire
→ 3 phase 3 wire</p> <p>③ $V_L = \sqrt{3} V_{ph}$</p> <p>④ $I_L = I_{ph}$</p> <p>⑤ Line & Phase voltage is different</p> <p>⑥ Common point is called neutral / star point</p> <p>⑦ used for long distances</p> <p>⑧ often used in application which require less starting current</p> | <p>① 3 wire connection</p> <p>② only 3 phase 3 wire connection possible</p> <p>③ $V_L = V_{ph}$</p> <p>④ $I_L = \sqrt{3} I_{ph}$</p> <p>⑤ Line and Phase voltage are same.</p> <p>⑥ There is no neutral in delta connection</p> <p>⑦ used for short distances</p> <p>⑧ often used in application which require high starting torque</p> |
|--|---|

SOLVING STAR/DELTA NUMERICALS

- ① $Z_{ph} = \sqrt{R^2 + X_L^2}$ or $\sqrt{(\text{Real})^2 + (\text{imag})^2}$
- $X_L = \omega \times L$
 - $X_C = \frac{1}{\omega C}$
- ② $\phi = \tan^{-1} \left(\frac{X_L}{R} \right)$ degree imag / Real

③ $I_{ph} = \frac{V_{ph}}{Z_{ph}}$, Always given $\rightarrow V_L$

- For star $I_L = I_{ph}$, $V_L = \sqrt{3} V_{ph}$
- For delta $V_L = V_{ph}$, $I_L = \sqrt{3} I_{ph}$

④ Active 3 ϕ Power $P = V_L I_L \cos \phi \times \sqrt{3}$

Reactive $Q = \sqrt{3} \times V_L \times I_L \sin \phi$

Apparent $S = V_L \times I_L \times \sqrt{3}$

3 PHASE ACTIVE POWER

- $I_L = I_{ph}$, $3\phi P = 3 \times (I_{ph}^2 R)$
- $V_L = \sqrt{3} V_{ph}$
- $3\phi P = 3 \times (V_{ph} \times I_{ph} \times \cos \phi)$
- $\left(\frac{3 \times V_L \times I_L \cos \phi}{\sqrt{3}} \right)$
- $3\phi P = \sqrt{3} V_L I_L \cos \phi$ watt
- $3\phi Q = \sqrt{3} V_L I_L \sin \phi$ VAR
- $3\phi S = \sqrt{3} V_L I_L$ VA

- $\phi = \theta_a$
- $\frac{N_2}{N_1} = \frac{E_2}{E_1}$
- $\pm m$ equation, to find N_1
 $\rightarrow \frac{V_{out}}{V_{in}} = \frac{N_2}{N_1}$
- Now use Tx Ratio, get N_2
- Primary current
 $I_1 = \frac{VA}{V_1} \rightarrow I_A \times 10$